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Target Association Using Detection Methods

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A residual-based scheme is presented for solving the radar track-to-track association problem using bearings-only measurements. To accomplish track association between two stations, the residuals of a bank of nonlinear filters called modified gain extended Kalman filters are analyzed. Once tracks have been associated between two stations, tracks from additional stations may be associated with tracks from the first two stations by checking algebraic parity equations. Traditional track association methods rely on the local stations' estimated target positions and error variances. These local estimates may be quite inaccurate or even divergent when using bearings-only measurements. Our method bypasses this difficulty because our filters use raw data from multiple stations. An example demonstrates that our methods yield results superior to those of standard methods.

I. Introduction

SUPPOSE that several spatially distributed radar installations are each tracking several targets. Associating a given target to its track at each of the radar stations is an important issue, which the radar literature refers to as the track-to-track association problem. Suppose further that the stations use passive sensors that only measure bearings to the target, without measuring range. In this paper, we outline a strategy for solving this association problem by analyzing measurement residuals.

Bearings-only observation functions fall into two special classes of nonlinear functions, called modifiable and approximately modifiable nonlinearities, which are defined as follows:

Definition 1. A time-varying function $f:\mathbb{R}^n \rightarrow \mathbb{R}^q$ is called modifiable if there exists an operator $A:\mathbb{R}^q \times \mathbb{R}^n \rightarrow \mathbb{R}^{q\times n}$ such that for any $x, \bar{x} \in \mathbb{R}^n$,

$$f(x) - f(\bar{x}) = A[f(x), \bar{x}](x-\bar{x}) \tag{1}$$

Definition 2. A time-varying function $f:\mathbb{R}^n \rightarrow \mathbb{R}^q$ is called approximately modifiable if there exists a region $\mathcal{D} \subset \mathbb{R}^n$ and operators $A:\mathbb{R}^q \times \mathbb{R}^n \rightarrow \mathbb{R}^{q\times n}$ and $\mathcal{E}:\mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^{n\times n}$ such that for any x, $\bar{x} \in \mathcal{D}$,

$$f(x) - f(\bar{x}) = [A(f(x),\bar{x}) + \mathcal{E}(x, x-\bar{x})](x-\bar{x}) \tag{2}$$

where $\lim_{\|x-\bar{x}\|\rightarrow 0} \|\mathcal{E}(x, x-\bar{x})\|/\|A(f(x),\bar{x})\| = 0$.

Song and Speyer's modified gain extended Kalman filter (MGEKF)[1] is a globally convergent, unbiased, nonlinear observer for systems whose measurement functions are modifiable or approximately modifiable. In this paper, the observers we design for bearings-only track association are MGEKFs.

An early attempt at solving the track-to-track association problem was made by Singer and Kanyuck.[2] In their paper, they incorrectly assumed that estimation errors local to each station were uncorrelated. Bar-Shalom,[3] Bar-Shalom and Fortmann,[4] and Bar-Shalom and Campo[5] later corrected this error by accounting for the correlation between the local estimation errors due to the common process noise of the target. Later researchers have integrated the problem of track association directly into the process of separating the measurements corresponding to actual targets from clutter.[6,7] In all of these references, it is assumed that both range and bearings were measured. In some of these references, the possibility of using a MGEKF to handle the situation of bearings-only measurements is mentioned, but none have a discussion of the details of such an implementation, in particular problems associated with the asymmetry of single station estimation errors. Estimates based on bearings-only measurements from a single station are especially uncertain along the line between the target and the receiver. This uncertainty is reduced when measurements from physically separated stations are used. Our method attempts to take advantage of this phenomenon by using estimates constructed from several stations' measurements.

The paper is organized as follows. We show in Sec. II that bearings-only measurement functions are modifiable. (Prior results only showed that they were approximately modifiable.[1]) We then demonstrate in Sec. III that incorrect associations between two radar stations can be interpreted as sensor faults, so that a bank of modified-gain fault detection filters can be used to determine the track associations. Section IV contains the main result, an algorithm for solving the bearings-only track association problem. The application of this algorithm to an example in Sec. V compares our approach to a conventional track association method. Section VI concludes the paper.

In the sequel, inertial Cartesian coordinates describe the motion of each target in three dimensions via the state vector

$$\boldsymbol{x}^t = [X^t \quad Y^t \quad Z^t \quad \dot{X}^t \quad \dot{Y}^t \quad \dot{Z}^t \quad \ddot{X}^t \quad \ddot{Y}^t \quad \ddot{Z}^t]^T \tag{3}$$

and the dynamics of each target are assumed to be of the form

$$\boldsymbol{x}^t(k+1) = \boldsymbol{A}(k)\boldsymbol{x}^t(k) + \boldsymbol{B}(k)\boldsymbol{w}^t(k) \tag{4}$$

Note that we include an acceleration state to model maneuvering target dynamics.

II. Modifiability of Bearings-Only Measurements

Song and Speyer[1] showed that the azimuth angle $az_s^t \in [-\pi/2, \pi/2)$ and the elevation angle $el_s^t \in [-\pi/2, \pi/2)$ from station s to target t, as shown in Fig. 1, are modifiable and approximately modifiable, respectively. The region $\mathcal{D}$ in which the elevation angle was approximately modifiable excluded an ellipsoidal region near the sensor, making their algorithms difficult to implement for situations where the angular sensor gets close to the target, for example, in the terminal guidance of a missile. We improve this situation somewhat by introducing the new angle $\Psi_s^t \in [-\pi/2, \pi/2)$ and describing the position of the target in terms of Ψ_s^t and $\Phi_s^t \triangleq az_s^t$. Note that Ψ_s^t can be calculated from az_s^t and el_s^t via the equation

$$\Psi_s^t = \tan^{-1}\left(\frac{Z_s^t}{X_s^t}\right) = \tan^{-1}\left(\frac{\tan el_s^t}{\cos az_s^t}\right) \tag{5}$$

This section is devoted to proving that the measurement function for Ψ_s^t is modifiable.

Let $\bar{\boldsymbol{x}}^t$ be an estimate of $\boldsymbol{x}^t$ and assume that the position of the measurement station in inertial space, $\boldsymbol{x}_s = [X_s \quad Y_s \quad Z_s]$, is known.

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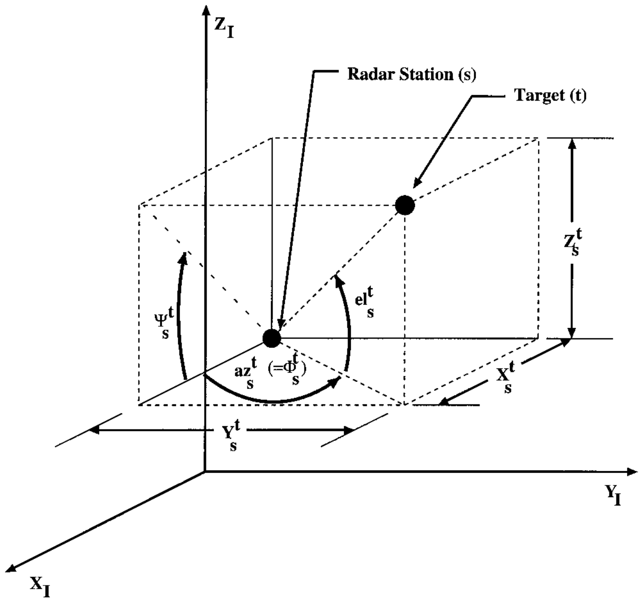

Fig. 1 Angles for target bearings.

Then X_s^t, Y_s^t, Z_s^t, $\bar{X}_s^t$, $\bar{Y}_s^t$, and $\bar{Z}_s^t$ can be computed by taking the difference between elements of $\boldsymbol{x}^t$, $\bar{\boldsymbol{x}}^t$, and $\boldsymbol{x}_s$.

Suppose that station s measures the bearings of target t with the measurement vector z_s^t. Define $\boldsymbol{h}_s(\boldsymbol{x}^t)$ by

$$\boldsymbol{h}_s(\boldsymbol{x}^t) \triangleq \begin{bmatrix} \Phi_s^t \\ \Psi_s^t \end{bmatrix} = z_s^t \tag{6}$$

The measurement residual corresponding to $\boldsymbol{h}_s(\boldsymbol{x}^t)$ is then

$$\boldsymbol{h}_s(\boldsymbol{x}^t) - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^t) = \begin{bmatrix} \tan^{-1}\left(Y_s^t/X_s^t\right) - \tan^{-1}\left(\bar{Y}_s^t/\bar{X}_s^t\right) \\ \tan^{-1}\left(Z_s^t/X_s^t\right) - \tan^{-1}\left(\bar{Z}_s^t/\bar{X}_s^t\right) \end{bmatrix} \triangleq \begin{bmatrix} \tan^{-1}\alpha \\ \tan^{-1}\beta \end{bmatrix} \tag{7}$$

Applying the trigonometric identity

$$\tan^{-1}(a) - \tan^{-1}(b) = \tan^{-1}[(a-b)/(1+ab)]$$

we obtain

$$\begin{bmatrix} \tan^{-1}\alpha \\ \tan^{-1}\beta \end{bmatrix} = \begin{bmatrix} \tan^{-1} \dfrac{\left(Y_s^t/X_s^t\right) - \left(\bar{Y}_s^t/\bar{X}_s^t\right)}{1 + \left(Y_s^t/X_s^t\right)\left(\bar{Y}_s^t/\bar{X}_s^t\right)} \\ \tan^{-1} \dfrac{\left(Z_s^t/X_s^t\right) - \left(\bar{Z}_s^t/\bar{X}_s^t\right)}{1 + \left(Z_s^t/X_s^t\right)\left(\bar{Z}_s^t/\bar{X}_s^t\right)} \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \tan^{-1}\alpha \\ \tan^{-1}\beta \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\dfrac{Y_s^t\bar{X}_s^t - \bar{Y}_s^t X_s^t}{X_s^t\bar{X}_s^t + Y_s^t\bar{Y}_s^t}\right) \\ \tan^{-1}\left(\dfrac{Z_s^t\bar{X}_s^t - \bar{Z}_s^t X_s^t}{X_s^t\bar{X}_s^t + Z_s^t\bar{Z}_s^t}\right) \end{bmatrix} \tag{9}$$

Define

$$\boldsymbol{H}\left(z_s^t\right) \triangleq \begin{bmatrix} \sin\left(\Phi_s^t\right) & -\cos\left(\Phi_s^t\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sin\left(\Psi_s^t\right) & 0 & -\cos\left(\Psi_s^t\right) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

Let $d_1 \triangleq \sqrt{[(X_s^t)^2 + (Y_s^t)^2]}$, $D_1 \triangleq d_1/[X_s^t\bar{X}_s^t + Y_s^t\bar{Y}_s^t]$, $d_2 \triangleq \sqrt{[(X_s^t)^2 + (Z_s^t)^2]}$, and $D_2 \triangleq d_2/[X_s^t\bar{X}_s^t + Z_s^t\bar{Z}_s^t]$. Note also that $\sin(\Phi_s^t) = Y_s^t/d_1$, $\cos(\Phi_s^t) = X_s^t/d_1$, $\sin(\Psi_s^t) = Z_s^t/d_2$, and $\cos(\Psi_s^t) = X_s^t/d_2$. Therefore, we can express D_1 and D_2 as functions of the estimates and measured angles:

$$D_1 = D_1\left(z_s^t, \bar{\boldsymbol{x}}^t\right) = 1\big/\left[\cos\left(\Phi_s^t\right)\bar{X}_s^t + \sin\left(\Phi_s^t\right)\bar{Y}_s^t\right]$$

$$D_2 = D_2\left(z_s^t, \bar{\boldsymbol{x}}^t\right) = 1\big/\left[\cos\left(\Psi_s^t\right)\bar{X}_s^t + \sin\left(\Psi_s^t\right)\bar{Z}_s^t\right]$$

If we express the trigonometric functions in $\boldsymbol{H}(z_s^t)$, D_1, and D_2 in terms of X_s^t, Y_s^t, Z_s^t, $\bar{X}_s^t$, $\bar{Y}_s^t$, and $\bar{Z}_s^t$, we can write Eq. (9) as a function of z_s^t and $\bar{\boldsymbol{x}}^t$:

$$\begin{bmatrix} \alpha\left(z_s^t, \bar{\boldsymbol{x}}^t\right) \\ \beta\left(z_s^t, \bar{\boldsymbol{x}}^t\right) \end{bmatrix} = \begin{bmatrix} D_1\left(z_s^t, \bar{\boldsymbol{x}}^t\right) & 0 \\ 0 & D_2\left(z_s^t, \bar{\boldsymbol{x}}^t\right) \end{bmatrix} \boldsymbol{H}\left(z_s^t\right)\left[\bar{\boldsymbol{x}}^t - \boldsymbol{x}_s\right] \tag{11}$$

Finally, we can rewrite Eq. (11) as

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/\alpha\left(z_s^t, \bar{\boldsymbol{x}}^t\right) & 0 \\ 0 & 1/\beta\left(z_s^t, \bar{\boldsymbol{x}}^t\right) \end{bmatrix} \begin{bmatrix} D_1\left(z_s^t, \bar{\boldsymbol{x}}^t\right) & 0 \\ 0 & D_2\left(z_s^t, \bar{\boldsymbol{x}}^t\right) \end{bmatrix} \times \boldsymbol{H}\left(z_s^t\right)\left[\boldsymbol{x}^t - \boldsymbol{x}^t - \bar{\boldsymbol{x}}^t + \boldsymbol{x}_s\right] \tag{12}$$

and combine it with Eq. (7) to obtain $\boldsymbol{h}_s(\boldsymbol{x}^t)$ in modifiable form,

$$\boldsymbol{h}_s(\boldsymbol{x}^t) - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^t) = \begin{bmatrix} -\dfrac{D_1\left(z_s^t, \bar{\boldsymbol{x}}^t\right)\tan^{-1}\alpha\left(z_s^t, \bar{\boldsymbol{x}}^t\right)}{\alpha\left(z_s^t, \bar{\boldsymbol{x}}^t\right)} & 0 \\ 0 & -\dfrac{D_2\left(z_s^t, \bar{\boldsymbol{x}}^t\right)\tan^{-1}\beta\left(z_s^t, \bar{\boldsymbol{x}}^t\right)}{\beta\left(z_s^t, \bar{\boldsymbol{x}}^t\right)} \end{bmatrix} \times \boldsymbol{H}\left(z_s^t\right)[\boldsymbol{x}^t - \bar{\boldsymbol{x}}^t] \tag{13}$$

where we have made use of the identity

$$\boldsymbol{H}\left(z_s^t\right)[\boldsymbol{x}_s - \boldsymbol{x}^t] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus, we have replaced the elevation angle el_s^t, from which Song and Speyer[1] produced an approximately modifiable function with a new angle Ψ_s^t. Like the azimuth angle Φ_s^t, angle Ψ_s^t leads to modifiable measurement functions.

III. Converting Incorrect Associations into Sensor Faults

Suppose that station s can view several targets, indexed by i, and measures the bearings of each target. Then each of these measurements z_s^i is generated by $\boldsymbol{h}_s(\boldsymbol{x}^i)$, as in Eq. (6). Now suppose that another station, using its local observations, generates a state estimate of one of the targets that station s views. This estimate $\bar{\boldsymbol{x}}^j$ corresponds to $\boldsymbol{x}^j$, the true state of the jth target at station s, but neither station knows the value of index j. Our goal is to determine which of the tracks at station s is the jth one, using only $\{z_s^i\}$, the measurements local to station s, and $\bar{\boldsymbol{x}}^j$, the other station's state estimate of one of the targets.

To this end, let us form the following error residual between the estimate $\bar{\boldsymbol{x}}^j$ and the measurement z_s^i, making use of the result from the preceding section:

$$z_s^i - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^j) = \boldsymbol{h}_s(\boldsymbol{x}^i) - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^j) = \boldsymbol{G}\left(z_s^i, \bar{\boldsymbol{x}}^j\right)(\boldsymbol{x}^i - \bar{\boldsymbol{x}}^j) \tag{14}$$

where from Eq. (13)

$$\boldsymbol{G}\left(z_s^i, \bar{\boldsymbol{x}}^j\right) = \begin{bmatrix} -\dfrac{D_1\left(z_s^i, \bar{\boldsymbol{x}}^j\right)\tan^{-1}\alpha\left(z_s^i, \bar{\boldsymbol{x}}^j\right)}{\alpha\left(z_s^i, \bar{\boldsymbol{x}}^j\right)} & 0 \\ 0 & -\dfrac{D_2\left(z_s^i, \bar{\boldsymbol{x}}^j\right)\tan^{-1}\beta\left(z_s^i, \bar{\boldsymbol{x}}^j\right)}{\beta\left(z_s^i, \bar{\boldsymbol{x}}^j\right)} \end{bmatrix} \times \boldsymbol{H}\left(z_s^i\right) \tag{15}$$

By introducing a zero term into the measurement residual, we can rephrase it as

$$z_s^i - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^j) = \boldsymbol{h}_s(\boldsymbol{x}^i) - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^j) \tag{16}$$

$$z_s^i - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^j) = \boldsymbol{G}\big(z_s^i, \bar{\boldsymbol{x}}^j\big)(\boldsymbol{x}^i - \bar{\boldsymbol{x}}^j) \tag{17}$$

$$z_s^i - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^j) = \boldsymbol{G}\big(z_s^i, \bar{\boldsymbol{x}}^j\big)(\boldsymbol{x}^i - \bar{\boldsymbol{x}}^j + \boldsymbol{x}^j - \boldsymbol{x}^j) \tag{18}$$

$$z_s^i - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^j) = \boldsymbol{G}\big(z_s^i, \bar{\boldsymbol{x}}^j\big)(\boldsymbol{x}^j - \bar{\boldsymbol{x}}^j) + \boldsymbol{G}\big(z_s^i, \bar{\boldsymbol{x}}^j\big)(\boldsymbol{x}^i - \boldsymbol{x}^j) \tag{19}$$

$$z_s^i - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^j) = \boldsymbol{G}\big(z_s^i, \bar{\boldsymbol{x}}^j\big)(\boldsymbol{x}^j - \bar{\boldsymbol{x}}^j) + \boldsymbol{\mu}_s^{ij} \tag{20}$$

where $\boldsymbol{\mu}_s^{ij} \triangleq \boldsymbol{G}(z_s^i, \bar{\boldsymbol{x}}^j)(\boldsymbol{x}^i - \boldsymbol{x}^j)$ represents the difference between $\boldsymbol{x}^i$ and $\boldsymbol{x}^j$ as a sensor fault. If $i = j$, we have correctly guessed the association between measurement and estimate, and there is no fault ($\boldsymbol{\mu}_s^{ij} = 0$). If $i \neq j$, then $\boldsymbol{\mu}_s^{ij} \neq 0$, playing the role of a sensor fault in the residual.

IV. Algorithm for Track Association from Bearings-Only Measurements via Fault Detection Filters

Suppose that there are S radar stations, with known inertial coordinates, that make bearings-only measurements in three-space of T different targets. We assume that all measurements at each station have been grouped as tracks of each target visible at that station using conventional means.[4,8,9] In this section, we propose an algorithm for associating the tracks at all stations to their corresponding targets.

Assume that each measurement station s is located at known inertial coordinates (X_s, Y_s, Z_s). Let $\bar{\boldsymbol{x}}^{1i}$ denote a fault detection filter's estimate of the target corresponding to the ith track at the first station. The bearings-only measurement function for the station s of the same target is thus

$$\boldsymbol{h}_s(\boldsymbol{x}^{1i}) \triangleq \begin{bmatrix} \tan^{-1}\left(\dfrac{Y^{1i} - Y_s}{X^{1i} - X_s}\right) \\[2ex] \tan^{-1}\left(\dfrac{Z^{1i} - Z_s}{X^{1i} - X_s}\right) \end{bmatrix}$$

From the results of the preceding section, the error residual of track j at any station s, generated by target i at station 1, is given by

$$z_s^j - \boldsymbol{h}_s(\bar{\boldsymbol{x}}^{1i}) \approx \boldsymbol{G}\big(z_s^j, \bar{\boldsymbol{x}}^{1i}\big)(\boldsymbol{x}^{1i} - \bar{\boldsymbol{x}}^{1i}) + \boldsymbol{\mu}_s^{ij} + \boldsymbol{v}^j \tag{21}$$

where $\boldsymbol{G}(z_s^j, \bar{\boldsymbol{x}}^{1i})$ is given by Eq. 15 and the sensor noise is

$$\boldsymbol{v}^j = \mathcal{N}(0, \boldsymbol{V}^j)$$

The approximate structure of Eq. (21) is due to the replacement of the measurement function in $\boldsymbol{G}(\cdot, \cdot)$ with the actual measurement (see Song and Speyer[1]). Note that, by default, $\boldsymbol{\mu}_1^{ii} = 0$, $\forall i = 1, \ldots, T$.

The following algorithm, illustrated in Fig. 2, associates tracks between stations.

Algorithm (track association):

1) Let $i = 1$.

2) Run a bank of T detection filters that operate on data from stations 1 and 2, where the jth filter attempts to detect $\boldsymbol{\mu}_2^{ij}$. Each filter is constructed using the dynamic detection filter procedure given next. All but one of these detection filters should register a fault. The track corresponding to the filter that detected no fault is associated with z_1^i. Without loss of generality, label this track z_2^i.

3) For each track z_s^l, $s = 3, \ldots, S$, $l = 1, \ldots, T$, perform the algebraic parity test given subsequently. If the result of the parity test is zero, then z_s^l is associated with z_1^i and z_2^i.

4) If $i < T$, increment i by 1 and go to step 2. If $i = T$, we have completed the track association procedure.

Note that estimates obtained in step 2 are used in step 3. Therefore, stations 1 and 2 should be chosen to maximize observability of the targets.

Dynamic Detection Filter

For any estimator of $\boldsymbol{x}^{1i}$, the estimation residual determined by the measurements z_1^i and z_2^j will not converge to values near zero unless z_1^i and z_2^j correspond to the same target. One such estimator is the MGEKF[1] given as

$$\bar{\boldsymbol{x}}^{1i}(k+1) = \boldsymbol{A}(k)\hat{\boldsymbol{x}}^{1i}(k) \tag{22}$$

$$\boldsymbol{r}^{ij}(k) = \begin{Bmatrix} z_1^i(k) - \boldsymbol{h}_1[\hat{\boldsymbol{x}}^{1i}(k)] \\ z_2^j(k) - \boldsymbol{h}_2[\hat{\boldsymbol{x}}^{1i}(k)] \end{Bmatrix} \tag{23}$$

$$\hat{\boldsymbol{x}}^{1i}(k) = \bar{\boldsymbol{x}}^{1i}(k) + \boldsymbol{K}^{ij}(k)\boldsymbol{r}^{ij}(k) \tag{24}$$

$$\boldsymbol{M}^{ij}(k+1) = \boldsymbol{A}(k)\boldsymbol{P}^{ij}(k)\boldsymbol{A}^T(k) + \boldsymbol{Q}(k) \tag{25}$$

$$\bar{\boldsymbol{h}}_{x^{1i}(k)} = \begin{bmatrix}
-\dfrac{\bar{Y}^{1i} - Y_1}{\left\{1 + \left[\left(\bar{Y}^{1i} - Y_1\right)/\left(\bar{X}^{1i} - X_1\right)\right]^2\right\}\left(\bar{X}^{1i} - X_1\right)^2} & \dfrac{1}{\left\{1 + \left[\left(\bar{Y}^{1i} - Y_1\right)/\left(\bar{X}^{1i} - X_1\right)\right]^2\right\}\left(\bar{X}^{1i} - X_1\right)^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\[3ex]
-\dfrac{\bar{Z}^{ij} - Z_1}{\left\{1 + \left[\left(\bar{Z}^{ij} - Z_1\right)/\left(\bar{X}^{1i} - X_1\right)\right]^2\right\}\left(\bar{X}^{1i} - X_1\right)^2} & 0 & \dfrac{1}{\left\{1 + \left[\left(\bar{Z}^{ij} - Z_1\right)/\left(\bar{X}^{1i} - X_1\right)\right]^2\right\}\left(\bar{X}^{1i} - X_1\right)^2} & 0 & 0 & 0 & 0 & 0 & 0 \\[3ex]
-\dfrac{\bar{Y}^{1i} - Y_2}{\left\{1 + \left[\left(\bar{Y}^{1i} - Y_2\right)/\left(\bar{X}^{1i} - X_2\right)\right]^2\right\}\left(\bar{X}^{1i} - X_2\right)^2} & \dfrac{1}{\left\{1 + \left[\left(\bar{Y}^{1i} - Y_2\right)/\left(\bar{X}^{1i} - X_2\right)\right]^2\right\}\left(\bar{X}^{1i} - X_2\right)^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\[3ex]
-\dfrac{\bar{Z}^{ij} - Z_2}{\left\{1 + \left[\left(\bar{Z}^{ij} - Z_2\right)/\left(\bar{X}^{1i} - X_2\right)\right]^2\right\}\left(\bar{X}^{1i} - X_2\right)^2} & 0 & \dfrac{1}{\left\{1 + \left[\left(\bar{Z}^{ij} - Z_2\right)/\left(\bar{X}^{1i} - X_2\right)\right]^2\right\}\left(\bar{X}^{1i} - X_2\right)^2} & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{26}$$

Table 1 Radar station positions

Station identification	x Position, m	y Position, m	z Position, m
1	50	1	0
2	50,000	1	50
3	25,000	−400	100

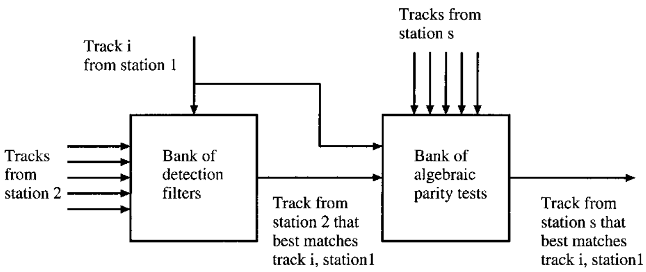

Fig. 2 Track association procedure.

$$\boldsymbol{K}^{ij}(k) = \boldsymbol{M}^{ij}(k)\bar{\boldsymbol{h}}^T_{x^{1i}(k)}\left[\bar{\boldsymbol{h}}_{x^{1i}(k)}\boldsymbol{M}^{ij}(k)\bar{\boldsymbol{h}}^T_{x^{1i}(k)} + \boldsymbol{V}^{ij}(k)\right]^{-1} \tag{27}$$

$$\tilde{\boldsymbol{G}}\left[z_1^i(k), z_2^j(k), \bar{\boldsymbol{x}}^{1i}(k)\right] = \begin{bmatrix} \boldsymbol{G}\left(z_1^i(k), \bar{\boldsymbol{x}}^{1i}(k)\right) \\ \boldsymbol{G}\left(z_2^j(k), \bar{\boldsymbol{x}}^{1i}(k)\right) \end{bmatrix} \tag{28}$$

$$\begin{aligned} \boldsymbol{P}^{ij}(k) = {} & \left\{\boldsymbol{I} - \boldsymbol{K}^{ij}(k)\tilde{\boldsymbol{G}}\left[z_1^i(k), z_2^j(k), \bar{\boldsymbol{x}}^{1i}(k)\right]\right\}\boldsymbol{M}^{ij}(k) \\ & \times \left\{\boldsymbol{I} - \boldsymbol{K}^{ij}(k)\tilde{\boldsymbol{G}}\left[z_1^i(k), z_2^j(k), \bar{\boldsymbol{x}}^{1i}(k)\right]\right\}^T \\ & + \boldsymbol{K}^{ij}(k)(\boldsymbol{V}^{ij})^{-1}(k)(\boldsymbol{K}^{ij})^T(k) \end{aligned} \tag{29}$$

where

$$\boldsymbol{V}^{ij}(k) = \mathrm{diag}\{\boldsymbol{V}^i, \boldsymbol{V}^j\} \tag{30}$$

The weighted innovations process of the MGEKF,

$$\boldsymbol{\nu}^{ij}(k) = \left[\bar{\boldsymbol{h}}_{x^{1i}(k)}\boldsymbol{M}^{ij}(k)\bar{\boldsymbol{h}}^T_{x^{1i}(k)} + \boldsymbol{V}^{ij}(k)\right]^{\frac{1}{2}}\boldsymbol{r}^{ij}(k) \tag{31}$$

should be close to a zero-mean, unit variance white noise sequence only if z_1^i and z_2^j correspond to the same target.

Algebraic Parity Test

This test determines if z_s^l, $S \geq s > 2$, $T \geq l \geq 1$, is associated with z_1^i and z_2^i, where z_1^i and z_2^i are already known to be associated with each other. Suppose that $\hat{\boldsymbol{x}}^{1i}$ is the state estimate generated by z_1^i and z_2^i. Then, if z_s^l is associated with the tracks z_1^i and z_2^i,

$$\boldsymbol{\nu}(k)_l^{1i} \triangleq \left[\tilde{\boldsymbol{h}}_{x^{1i}(k)}\boldsymbol{M}^{ij}(k)\tilde{\boldsymbol{h}}^T_{x^{1i}(k)} + \boldsymbol{V}^{ij}(k)\right]^{\frac{1}{2}}\left\{z_s^l(k) - \boldsymbol{h}_s[\hat{\boldsymbol{x}}^{1i}(k)]\right\} \tag{32}$$

should be close to a zero mean, unit variance white noise sequence. Here, the approximate measurement matrix $\tilde{\boldsymbol{h}}_{x^{1i}(k)}$ is computed in a manner similar to the first two rows of the matrix in Eq. (26), but referenced to (X_s, Y_s, Z_s), the location of station s, instead of the location of the first station (X_1, Y_1, Z_1). The algebraic parity test is simply to evaluate the parity equation (32).

V. Example

The track association algorithm presented in the last section is applied to simulation data in this section. Three radar installations were located at the positions given by Table 1, and two targets were both modeled as ninth-order linear time-invariant discrete-time systems with the dynamics

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}\boldsymbol{x}(t) + \Gamma\boldsymbol{w}(t) \tag{33}$$

where

$$\boldsymbol{F} \triangleq \begin{bmatrix} 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&-\alpha&0&0 \\ 0&0&0&0&0&0&0&-\alpha&0 \\ 0&0&0&0&0&0&0&0&-\alpha \end{bmatrix}$$

$$\Gamma \triangleq \begin{bmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix} \tag{34}$$

and where $\boldsymbol{w}$ is a zero mean Brownian motion process with covariance $\boldsymbol{I}_{3\times 3}$ and $\alpha = \frac{1}{10}$ is the time constant for the first-order filters that model target maneuvers as colored noise processes. We sample this model at intervals of $T = 0.1$ s to generate the discrete time dynamics

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{w}(k) \tag{35}$$

where

$$\boldsymbol{A} = e^{FT}, \qquad \boldsymbol{B} = \int_0^T e^{Ft}B\,\mathrm{d}t, \qquad E[\boldsymbol{w}(k)] = \boldsymbol{0}_{3\times 1}$$

$$E[\boldsymbol{w}(k)\boldsymbol{w}^T(l)] = \boldsymbol{I}_{3\times 3}\delta_{kl} \tag{36}$$

The targets began the simulation with the initial conditions

$$\boldsymbol{x}_1(0) = [50 \quad 220{,}000 \quad 30{,}000 \quad 250 \quad -1000 \quad 0 \quad 0 \quad 0 \quad 0]^T$$

$$\boldsymbol{x}_2(0) = [50{,}000 \quad 20{,}000 \quad 35{,}000 \quad -250 \quad 1000 \quad 0 \quad 0 \quad 0 \quad 0]^T$$

This configuration corresponds to the two targets initially moving directly toward each other, in a line that almost passes through station 2. In the simulation, they pass closest to each other at $t = 99.2$ s. Each measurement station measures the angles Φ_s^t and Ψ_s^t to each target at every sample time. These measurements are subject to additive, normally distributed zero-mean white measurement noise with standard deviation 1 deg. We assume that the measurement noise is independent between sensors at all stations. Each MGEKF begins with the a priori information

$$\hat{\boldsymbol{x}}^{1i}(0) = [25{,}000 \quad 120{,}000 \quad 32{,}500 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$

$$\boldsymbol{P}^{ij}(0) = 10^7 \times \boldsymbol{I}_{9\times 9}$$

Finally, we assume that the local stations were able to separate their measurements from clutter perfectly using methods like those of Reid[9] or Bar-Shalom and Fortmann,[4] or Fortmann and Bar-Shalom.[8]

Figure 3 plots the weighted innovations of a MGEKF that uses measurements from stations 1 and 2 that correspond to the second target, whereas Fig. 4 plots the weighted innovations of a MGEKF that uses measurements that are mismatched. Note that the innovations for the correct match appear to be a zero mean white noise sequence, whereas the innovations for the incorrect match are larger and are not white. To better observe the behavior of these sequences, their means were estimated using a Kalman filter (assuming that each element of the weighted innovation of the MGEKF was a measurement of a process that had integrator dynamics, process noise with

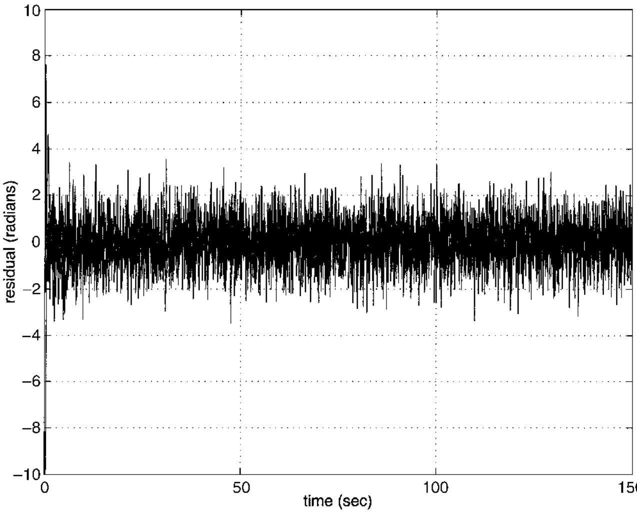

Fig. 3 MGEKF residual for matching tracks.

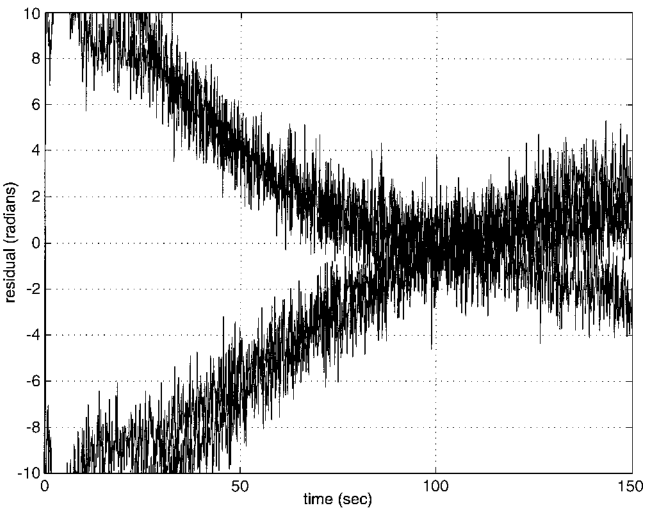

Fig. 4 MGEKF residual for mismatched tracks.

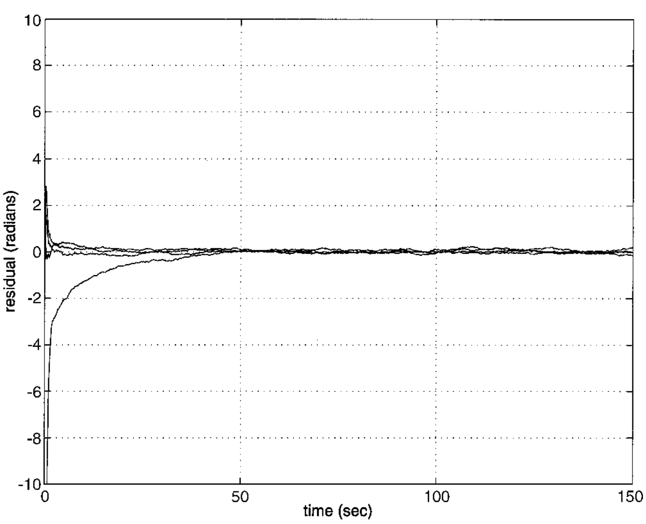

Fig. 5 Filtered MGEKF residual for matching tracks.

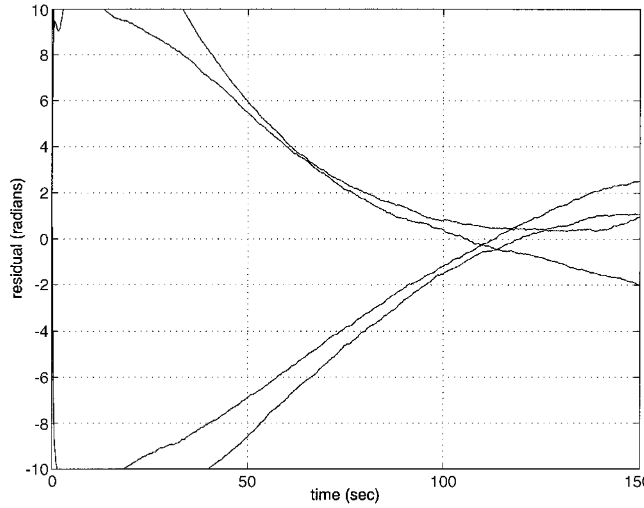

Fig. 6 Filtered MGEKF residual for mismatched tracks.

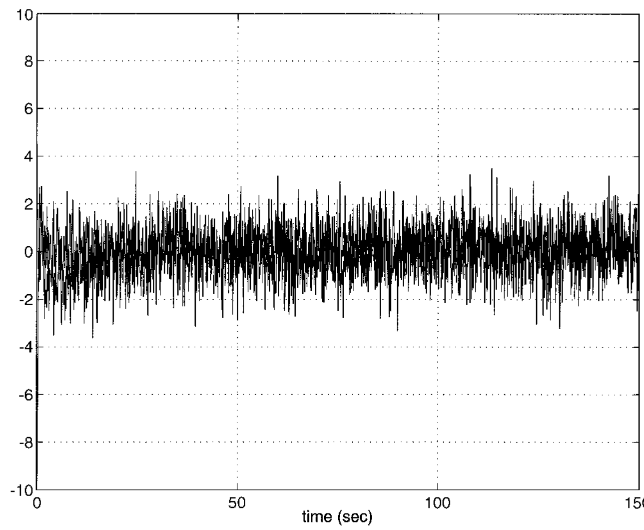

Fig. 7 Parity test residual for matching tracks.

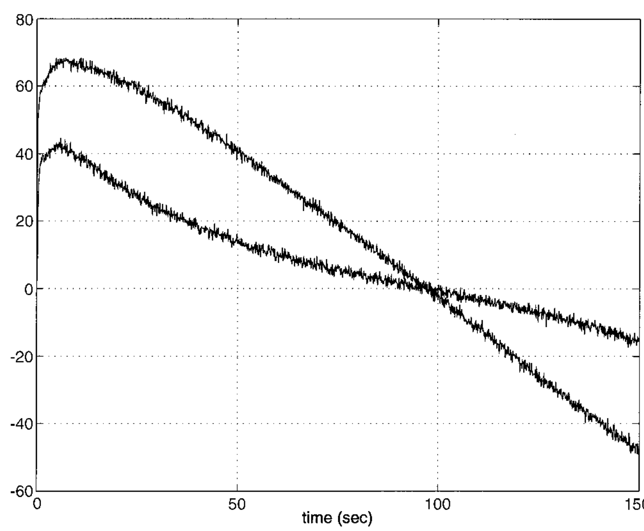

Fig. 8 Parity test residual for mismatched tracks.

covariance 10^{-3}, and measurement noise with covariance 1). These estimates (Figs. 5 and 6) clearly show that the mean corresponding to a mismatch looks nothing like that of the matched case.

After the tracks had been associated between the first two stations, algebraic parity tests attempted to associate the targets observed by the third station relative to those observed by the first and second stations. Two plots of residuals generated by the algebraic parity tests appear in Figs. 7 and 8. Again, the residuals for the mismatch are much larger than those corresponding to a correct association.

For purposes of comparison, Fig. 9 plots the error statistic developed by Bar-Shalom[3] and Bar-Shalom and Fortmann[4] for both a correct and an incorrect track association (using the same data sequences that were used by the filters in Figs. 3 and 4). Note that the chi-squared error statistic does not change much between the matched and mismatched cases. We also noticed that there

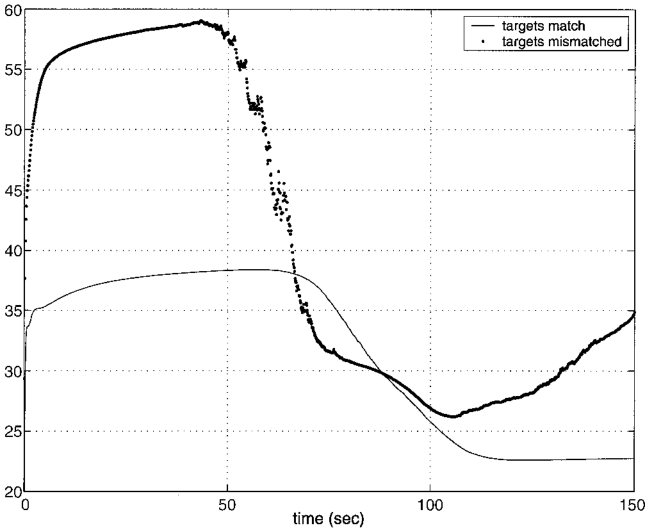

Fig. 9 Error statistic suggested by Bar-Shalom[3] and Bar-Shalom and Fortmann[4]: $(\hat{x}_1 - \hat{x}_2)^T E[(\hat{x}_1 - \hat{x}_2)(\hat{x}_1 - \hat{x}_2)^T](\hat{x}_1 - \hat{x}_2)$.

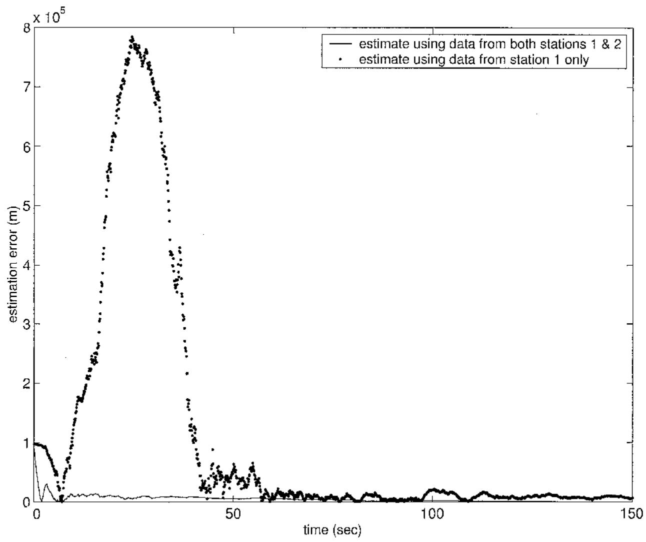

Fig. 10 Euclidean norm of error in tracking target 1.

were several instances where nearly singular matrices were inverted in the algorithm that computes the covariance of the difference between two local estimates.

Part of the reason for this difficulty is explained in Fig. 10, a plot of the Euclidean norm of the estimation error. The solid line corresponds to a MGEKF that uses measurements from both station 1 and 2, whereas the dotted line is from a filter that only used station 1 measurements. Any method that relies on estimates that only use a single station's measurements is subject to a large error. This is not a huge concern for linear estimators, but the matrix $\boldsymbol{P}^{ij}$ defined by Eq. (29) may not necessarily reflect this error.

We have also encountered cases where a single station measurement MGEKF was divergent in the radial direction to the target, but no such difficulties have appeared when data from two geographically disparate stations was used. One way of generating such a divergent case was to decrease the maneuver colored noise autocorrelation parameter α to $\frac{1}{20}$ or below. We note that values of this parameter below $\frac{1}{20}$ correspond to slower maneuvers, a commonly encountered situation.

VI. Conclusions

This paper describes residual-based techniques for solving the radar track association problem for bearings-only measurements. The association between the tracks at two stations can be determined by examining the residuals of a bank of MGEKFs. Once this association is established, an algebraic parity test can find the correspondence between tracks at other stations and targets tracked by the first two stations.

One may ask why detection filters are necessary: Why not do everything with algebraic parity tests? Although the detection filtering step is not strictly necessary, it does improve the quality of the track associations because the state estimates constructed from two widely separated stations are so much more accurate than the estimates from a single station.

To ensure the quality of the estimates from the MGEKFs, one could delay the algebraic parity testing steps for associating tracks from additional stations. If these parity tests are replaced with additional detection filter banks until the estimates before and after including a new station's measurements are sufficiently close, then the fidelity of the estimates can be guaranteed.

Acknowledgments

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